# Determination of Dielectric Constant and Loss Factor by Free Wave Method. II. Measurement on Water and Some Insulators at $\lambda = 9.8$ cm.

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## I. Measurement on Water by Reflection Method

Several attempts have been made to determine the dielectric constant and loss factor of a substance by the measurement of the reflection coefficient of a dielectric sheet placed on a metallic plate.[2] The experimental results cannot, however, be considered as sufficiently accurate, because of the incomplete method of calculating dielectric constant and loss factor from reflection coefficient and of the considerable experimental error which seems to arise mainly from the imperfect and unsteady source and the uneven metallic plate used as a reflector.

The former difficulty was already removed by one of us, as reported in the previous paper.[3] As to the latter, we have tried to remove it by constructing a steady and powerful source of wave and by using a mercury sheet as the reflector.

With an apparatus constructed in this way we have measured the dielectric constant and loss factor of water at the wave length of 9.8 cm. As will be reported in the following, water shows anomalous dispersion and absorption at this wave length.

The apparatus is shown schematically in Fig. 1. The 9.8 cm. wave generated with split-anode magnetron A was emitted vertically into the free space from a horn B through a wave guide G. A wooden vessel V with a rectangular surface of $60 \times 60$ cm.$^2$ was placed horizontally under B. This area was just sufficient to remove the edge effect. A crystal detector C consisting of silicon crystal and nickel wire was so adjusted that the deflection of microammeter attached to it was proportional to the energy of the wave. C was moved vertically between B and V, and the energy distribution of the standing wave was determined.

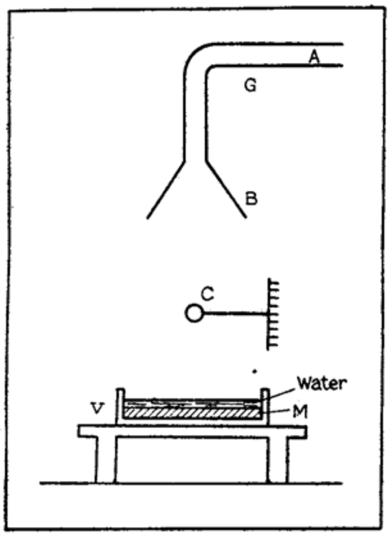


Fig. 1.

The measurement was carried out as follows. The vessel V is first filled with mercury M only. The incident wave with an amplitude $E_0$ is reflected by M to form a standing wave between B and M. The maximum and minimum values of the energy of the standing wave would be proportional to $4E_0^2$ and zero, respectively, when there would be no reflection other than that from the surface of the mercury. We could show that this was actually the case in our experiment.

Next, we place on the mercury a water layer whose thickness can be calculated accurately from the volume of water and the area of the vessel. The reflected and incident waves now form an imperfect standing wave, and the maximum and minimum values of its intensity will become proportional to $E_0^2(1+R)^2$ and $E_0^2(1-R)^2$, respectively. In other words, the ratio of the extreme values will be $(1-R)^2/(1+R)^2$. This ratio can be determined experimentally by moving C vertically and reading the microammeter deflections. The value of $R$ can at once be calculated from this ratio.

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(2) See *e.g.* G. Bäz, *Physik. Z.*, **40**, 394 (1939).

(3) M. Yasumi, this Bulletin, **24**, 53 (1951).

Such a measurement is made at different thickness $d$ of the water layer and finally a curve of $R^2$ *vs.* $d$ is obtained.

We can also determine the wave length in air from the positions of C corresponding to the extreme values of microammeter reading. The wave length thus determined was 9.8 cm. in agreement with the value obtained by the Lecher wire method.

The procedure of calculating dielectric constant $\varepsilon'$ and loss factor $\varepsilon''$ (or refractive index $n$ and absorption coefficient $k$) was reported in a previous paper.[3] The part necessary for the following discussion will be briefly repeated here.

The reflection coefficient of water layer of thickness $d$ placed on mercury is expressed as

$$R^2=\frac{\sinh^2\left(\frac{2\pi kd}{\lambda}+\frac{1}{2}\ln R_{12}\right)+\cos^2\left(\frac{2\pi nd}{\lambda}-\frac{\gamma}{2}\right)}{\sinh^2\left(\frac{2\pi kd}{\lambda}-\frac{1}{2}\ln R_{12}\right)+\cos^2\left(\frac{2\pi nd}{\lambda}+\frac{\gamma}{2}\right)} \tag{1}$$

where

$$R_{12}{}^2=\frac{(n-1)^2+k^2}{(n+1)^2+k^2} \tag{2}$$

$$\tan\gamma=\frac{2k}{(n^2-1)+k^2}\,. \tag{3}$$

For $n \gg k$, the maximum and minimum values $R_M{}^2$, $R_m{}^2$ of $R^2$ are calculated approximately as follows:

$$R_M \doteqdot \frac{R_{12}+e^{-\frac{4\pi kd_M}{\lambda}}}{1+R_{12}\,e^{-\frac{4\pi kd_M}{\lambda}}} \tag{4}$$

$$R_m \doteqdot \frac{R_{12}-e^{-\frac{4\pi kd_m}{\lambda}}}{1-R_{12}\,e^{-\frac{4\pi kd_m}{\lambda}}} \tag{5}$$

$$R_{12} \doteqdot \frac{n-1}{n+1} \tag{6}$$

$$d_M=\frac{2N\lambda}{4n} \tag{7}$$

$$d_m=\frac{(2N+1)\lambda}{4n} \tag{8}$$

$$(N=0,1,2,\ldots.)$$

where $d_M$ and $d_m$ are the values of $d$ corresponding to $R_M{}^2$ and $R_m{}^2$, respectively.

In the present experiment, $n$ and $k$ were obtained from the curve $R^2$ *vs.* $d$ in the following way. We found first an approximate value of $n$, using Eq. (7) or (8), and calculated approximate value of $R_{12}$, substituting the value of $n$ into Eq. (6). An approximate value of $k$ could be obtained from Eq. (4) or (5), using the said value of $R_{12}$, and $d_M$ or $d_m$. We drew a curve of $R^2$ *vs.* $d$, putting the values of $n$ and $k$ obtained above into Eq. (1) and compared it with the experimental curve. The calculated curve was slightly different from the experimental one, and we changed the values of $n$ and $k$ slightly until the former curve became almost identical with the latter. Thus the more exact values of $n$ and $k$ could be obtained.

The values of reflection coefficient $R^2$ of water at 5.5°C. obtained at various thicknesses $d$ of the layers are shown in Table 1. The values of $n$ and $k$ which are obtained by above-described procedure are shown in Table 2 which includes also those of the absorption index $\kappa(=k/n)$ and the real ($\varepsilon'$) and imaginary ($\varepsilon''$) parts of the complex dielectric constant.

Table 1

The Reflection Coefficient $R^2$ of the Water Layer of the Thickness $d$ (mm.) Measured at 5.5°C.

| $d$, mm. | $R^2$ | $d$, mm. | $R^2$ |
|---|---|---|---|
| 0.00 | 1.00 | 9.61 | 0.638 |
| 2.20 | 0.776 | 10.96 | 0.762 |
| 2.74 | 0.090 | 12.31 | 0.678 |
| 3.02 | 0.242 | 13.66 | 0.540 |
| 3.83 | 0.727 | 14.01 | 0.554 |
| 5.18 | 0.873 | 14.28 | 0.568 |
| 6.53 | 0.789 | 14.57 | 0.650 |
| 7.88 | 0.429 | 15.39 | 0.678 |
| 8.80 | 0.471 | | |

Table 2

Dielectric Data for Water

| | | | |
|---|---|---|---|
| $t$, °C. | 5.5 | $\kappa=(k/n)$ | 0.120 |
| $\lambda$, cm. | 9.8 | $\varepsilon'$ | 73.8 |
| $n$ | 8.65 | $\varepsilon''$ | 18.1 |
| $k$ | 1.04 | | |

The value of static dielectric constant of water at this temperature amounts to 86.1[4], which is much greater than the value of $\varepsilon'=73.8$ found in this experiment. Therefore water shows anomalous dispersion at this wave length. This is in conformity with the value of absorption index ($\kappa=0.120$) found by us. The detailed discussions on relaxation time, etc. will, however, be postponed until we obtain more data at different wave lengths.

(4) Calculated from the equation given by Drake, Pierce and Dow (*Phys. Rev.*, **35**, 613 (1930)).

$$\varepsilon_t=78.59[1-0.00461(t-25)+0.0000155(t-25)^2]$$

See also, Wymann, *Phys. Rev.*, **35**, 623 (1930).

## Measurement on Some Insulators by Transmission Method

The previous paper described the procedure of obtaining the dielectric constant and the loss factor of an insulating material from the transmission coefficient and the thickness of the sheet.(3) In the following we shall report some experimental data of transmission coefficient and the values of dielectric constant and loss factor obtained therefrom.

The apparatus is shown schematically in Fig. 2. As in the reflection experiment a linearly polarized wave of the wave-length of 9.8 cm. was generated from a split-anode magnetron A and was emitted vertical into the free space from a horn B through a wave guide G. A large aluminium plate P was placed at an angle of 45° to the floor to avoid the reflection of the wave from it. The fact that the reading of the microammeter connected to the crystal detector C remained constant in the middle part of the space between the horn and the aluminium plate showed that there was actually no reflection from the floor. As in the reflection experiment the detector system was so adjusted as to make the microammeter reading proportional to the intensity of the wave.

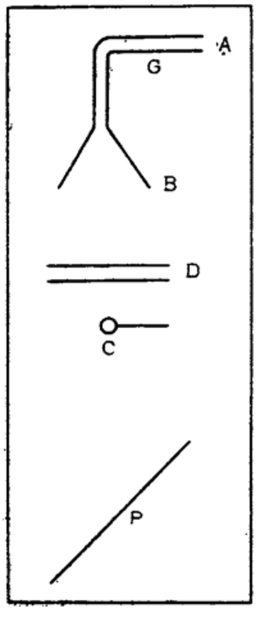


Fig. 2.

We now place a dielectric sheet D to be measured between the horn and the detector, and record the microammeter reading $I_d$ proportional to the energy of the transmitted wave. Then we remove the sheet and again record the microammeter reading $I_0$ proportional to the energy of the incident wave. The transmission coefficient of the sheet is simply given by

$$D^2 = I_d/I_0. \tag{9}$$

In this measurement we have, however, to show that in the former case the microammeter reading indicates only the energy of the transmitted wave. This was shown by the fact that the microammeter showed no deflection when an aluminium plate of the same size as the sample ($70\times70$ cm.$^2$) was placed upon the sample.

The transmission experiment was made on the sheets of ebonite, bakelite, methacryl resin, and polystyrene of various thicknesses $d$. The results are shown in Table 3.

Table 3

The Transmission Coefficient $D^2$ of Several Substances of the Thickness $d$ (mm.) Measured at the Wave-length of 9.8 cm.

| Ebonite 15°C. | | Bakelite 15°C. | | Methacryl resin 15°C. | | Polystyrene 5°C. | |
|---|---|---|---|---|---|---|---|
| $d$ | $D^2$ | $d$ | $D^2$ | $d$ | $D^2$ | $d$ | $D^2$ |
| 5 | 0.92 | 3 | 0.87 | 6 | 0.79 | 2 | 1.00 |
| 12 | 0.79 | 8 | 0.62 | 12 | 0.68 | 3.5 | 0.99 |
| 15 | 0.80 | 13 | 0.59 | 25 | 0.90 | 5 | 0.98 |
| 20 | 0.85 | 15 | 0.60 | 31 | 0.94 | 7 | 0.93 |
| 25 | 0.92 | 18 | 0.72 | | | 8.5 | 0.88 |
| 32 | 0.98 | 23 | 0.83 | | | | |
| 35 | 0.96 | 28 | 0.82 | | | | |

From these values of the transmission coefficient obtained at different thicknesses we can calculate refractive indices $n$, absorption coefficients $k$, dielectric constants $\varepsilon'$, and loss angles $\delta$ according to the procedure reported in the previous paper.(3) In Table 4 are shown the results of the calculation except that for polystyrene for which we have too few data to calculate these quantities.

Table 4

Dielectric Data for Ebonite, Bakelite and Methacryl Resin (15°C.)

| | Ebonite | Bakelite | Methacryl resin |
|---|---|---|---|
| $n$ | 1.60 | 1.92 | 1.65 |
| $\kappa=k/n$ | 0.005 | 0.035 | 0.015 |
| $\varepsilon'$ | 2.6 | 3.7 | 2.7 |
| $\tan\delta$ | 0.01 | 0.07 | 0.05 |

For these samples the condition $n \gg k$ holds and we can also use the approximate formulae derived by one of us.(3) Repeating them briefly,

$$D_M^2 = \frac{e^{-\frac{4\pi k d_M}{\lambda}}(1-R_{12}^2)^2}{(1-R_{12}^2 e^{-\frac{4\pi k d_M}{\lambda}})^2} \qquad (10)$$

$$D_m^2 = \frac{e^{-\frac{4\pi k d_m}{\lambda}}(1-R_{12}^2)^2}{(1+R_{12}^2 e^{-\frac{4\pi k d_m}{\lambda}})^2} \qquad (11)$$

$$R_{12} = \frac{n-1}{n+1} \qquad (6)$$

$$d_M = \frac{2N\lambda}{4n} \qquad (7)$$

$$d_m = \frac{(2N+1)\lambda}{4n} \qquad (8)$$

$$N = 0, 1, 2, 3, \ldots.$$

we have $D_M^2$ and $D_m^2$ are the maximum and the minimum values of $D^2$, $d_M$ and $d_m$ the corresponding values of thickness $d$, and $\lambda$ is wave length. Using Eq. (7) or Eq. (8), we first obtain the value of $n$ which we put into Eq. (6) and then calculate the value of $k$ from Eq. (10) for Eq. (11).


## Summary

The reflection coefficient of water layer placed on mercury has been measured at a wave length 9.8 cm. and at 5.5°C. From the curve of reflection coefficient *vs.* thickness of the water layer, dielectric constant and loss factor (or refractive index and absorption coefficient) have been calculated according to the procedure described previously.

Water has shown anomalous dispersion and absorption at this wave length.

The dielectric constant and loss factor have been measured for ebonite, bakelite and methacryl resin at the same wave length, by the transmission method.


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